

92 号 研 究 生 入 学 考 试 题

1. The von Karman integral equation for boundary layer flow over flat plate can be expressed as $\frac{\tau_w}{\rho} = \frac{d}{dx} \int_0^{\delta} v_x (v_{\infty} - v_x) dy$ where v_{∞} is the free stream velocity, τ_w is the wall shear stress and δ is the boundary layer thickness. If we assume that the velocity distribution in the boundary layer can be expressed as $\frac{v_x}{v_{\infty}} = a + b \sin(c\eta)$.
(a) Find a, b and c by using suitable boundary conditions (5%), (b) Find drag coefficient C_{Dx} as function of Reynolds number (10%).
2. Prove that the dimensionless velocity profile in the laminar sublayer in a turbulent flow can be expressed as $u^+ = y^+$. State all your assumptions in obtaining this relation. (10%)
3. The vapor pressure of water at 100°C is 101.3 kPa, explain why water will not boil at 100°C at sea level? (5%)
4. Assume there is a steel wall separating saturated steam condensing on one side ($h = 10,000 \text{ W/m}^2 \cdot \text{K}$), and air on the other side ($h = 50 \text{ W/m}^2 \cdot \text{K}$). If fins are to be added to the steel wall to increase heat transfer rate, which side should the fins be added to? Give explanation of your answer. (10%)
5. In analogy of momentum, energy and mass transfer, what are (a) Reynolds analogy and (b) Chilton-Colburn analogy? Specify under what conditions the analogies are valid. Explain all symbols you use. (10%)
6. In the anode chamber of a solid oxide fuel cell (SOFC), Fig. 5, flowing H_2 , N_2 and CO enter the chamber and react electrochemically with the oxygen which goes through the dense oxide membrane, and generates electricity like a battery. The two chemical reactions proceed according to the following equations;
$$\text{H}_2(\text{g}) + \text{O}(\text{s}) \rightarrow \text{H}_2\text{O}(\text{g}) \quad \text{O}(\text{s}) \text{ is the surface oxygen}$$
$$\text{CO}(\text{g}) + \text{O}(\text{s}) \rightarrow \text{CO}_2(\text{g})$$

Assume a layer of thickness δ adjacent to the anode represents the mass transfer resistance in the gas phase, the reactant H_2 diffuses through the layer at steady state. The boundary conditions are $z=0 \quad y_{\text{H}_2}=0, \quad z=\delta \quad y_{\text{H}_2}=y_0$.

(6a) Please set up the mass transfer governing equation for H_2 flux, and solve for the molar fraction of H_2 y_{H_2} and the molar flux of H_2 . (10%)

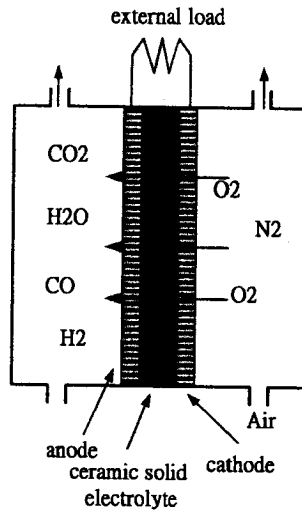
(6b) If the flowing gases are CH_4 , N_2 and only one chemical reaction occurs at the

anode,



Again, solve for the molar fraction of CH_4 y_{CH_4} and the molar flux of CH_4 .

The boundary conditions are $z=0$ $y_{\text{CH}_4}=0$, $z=\delta$ $y_{\text{CH}_4}=y_0$. (20%)



Solid Oxide Fuel Cell

Figure 5

7. A popular configuration of SOFC is an electrolyte “tube” coated with anode (inner) and cathode (outer) layer. The anode is a porous medium of thickness t . If we assume the mass transfer resistance is not in the gas phase, the mass transfer resistance is diffusion in the porous anode. When methane diffuses through the anode, it reacts with the surface oxygen, and the reaction rate is first order in methane concentration. Please solve the methane concentration profile in the anode layer. The boundary conditions are $r=R-t$ $C_m=C_0$, $r=R$ $C_m=0$. The diffusion coefficient of methane is D_{eff} . (20%)

You may need Bessel functions in solving the ordinary differential equation.

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} + \alpha^2 u = 0$$

$$u = A_1 Y_0(\alpha r) + A_2 J_0(\alpha r)$$