1. Lennard-Jones (6-12) potential 是常用於描述分子間距離變化能的關係式，它是由兩個項所組成，一個是 12 次方項，一個是 6 次方項，其中 12 次方項代表短距離作用力，6 次方項代表長距離作用力的作用貢獻。請問 12 次方項代表的長距離作用力是短距離作用力的幾倍？6 次方項呢？何者是吸引力？何者是斥力？（15%）

\[ \varphi(r) = 4 \varepsilon \left( \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right) \]

\( \sigma \): characteristic diameter of molecule (collision diameter)

\( \varepsilon \): characteristic energy of interaction

\( r \): distance between two molecules

\( \varphi \): interaction potential

2. 用柱狀媒質列文數，進行一階異質化学反應，同時擴散方程的方程式如下

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dC_a}{dr} \right) = k \alpha C_a \]

\( C_a = C_{a_0} \)

\( r \): radial position

\( \alpha \): diffusion coeff.

\( k \): reaction constant

請解出濃度分佈（Hint, 用 \( \frac{C_a}{C_{a_0}} = \frac{f(r)}{r} \) 代入）

3. 氣體之溫度是由其能量所定義的，所以其等氣體的溫度本應有許多個，例如 translational energy 計算所得之溫度，rotational energy 計算所得之溫度，vibrational energy 計算所得之溫度。但在一般條件下，此三者相等，這個溫度稱作平衡溫度。因爲氣體分子碰撞頻率極高，所以 translation \( \leftrightarrow \) rotation \( \leftrightarrow \) vibration mode 能量交換頻率，所以三者溫度相等。試討論在何种條件下，會有某一能量 mode 的溫度與其它不同的情形？（5%）
In a tubular hot-wall type chemical vapor deposition (CVD) reactor as shown in Fig. 4, ultrafine particles (particles with nanometer size) are formed in the gas phase at the inlet of the reactor and are transferred by the gas stream into the downstream of the reactor. During the transportation, the particles also diffuse to the reactor wall and deposit to form films. The problem of diffusion to the wall of a pipe from a laminar flow is formally identical with the heat transfer (Graetz) problem if the particle size is quite small compared with the pipe size. We can treat this problem by considering the steady-state equation of diffusion in cylindrical coordinates for a fully developed parabolic velocity profile as

\[
\frac{\partial N}{\partial x} = D_p \left[ \frac{\partial(r(\partial N/\partial r))}{\partial r} + \frac{\partial^2 N}{\partial x^2} \right] \quad (1)
\]

where \( u = 2u_{av}[1-(r/R)^2] \) and \( N \) is the particle concentration, \( D_p \) is the particle diffusion coefficient. The boundary conditions are

\[
N = N_0 \quad (r < R) \quad \text{when} \quad x = 0 \quad (2)
\]

\[
N = 0 \quad \text{when} \quad r = R \quad (3)
\]

where \( N_0 \) is the particle concentration at the beginning of diffusion and \( R \) is the radius of the reactor. If \( \partial^2 N/\partial x^2 \) can be neglected, Eq. (1) becomes

\[
u \frac{\partial N}{\partial x} = D_p \frac{\partial(r(\partial N/\partial r))}{\partial r} \]

(4)

Using the boundary conditions of Eqs. (2) and (3), the solution of Eq. (4) has the following form.

\[
\frac{N(r,x)}{N_0} = \sum_{i=0}^{\infty} K_i \Phi_i \left( \frac{x}{R} \right) \exp \left[ -\beta_i^2 \left( \frac{x}{R} \right) / Pe_R \right] \quad (5)
\]

where \( K_i \) are constants of integration, \( \beta_i \) are the eigenvalues, \( \Phi_i(r/R) \) are the corresponding eigenfunctions and \( Pe_R \) is the Peclet number defined as \( u_{av} (D_p/2R) \).

**Question 1:** Find the mass-transfer flux of the particles at the wall surface?

**Question 2:** If the ultrafine particles may stick on the reactor wall and form deposition and the deposition rate is proportional to the mass-transfer flux, then the deposition profile along the axial direction (x-axis) can be determined. Note that the eigenvalues and constants for these infinite series are given in Table 1. Under what kind of condition can only the first term \((I = 0)\) of great significance? Please check the axial (x-axis) position that can fulfill the quick convergence condition for \( u_{av} = 28.9 \, \text{m/s}, \, D_p = 2.03 \times 10^{-3} \, \text{m}^2/\text{s}, \, R = 3.35 \, \text{mm} \).
Table 1. Infinite-series solution functions for the tubular tube.

<table>
<thead>
<tr>
<th>$I$</th>
<th>$\beta_i^2$</th>
<th>$-(K_i/2)$</th>
<th>$\Phi_i'(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.312</td>
<td>0.749</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>44.62</td>
<td>0.544</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>113.8</td>
<td>0.463</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>215.2</td>
<td>0.414</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>348.5</td>
<td>0.382</td>
<td></td>
</tr>
</tbody>
</table>

$\Phi_i'(1)$ is the first order derivative of $\Phi_i(r/R)$ at $r = R$.

Fig. 4 Diffusion of untrufine particles in a tubular reactor.

\(25\%\) (4). A machine operator in a workshop complains that the air-heating system is not keeping the air at the required minimum temperature of 20 °C. To support his claim, he shows that a mercury-in-glass thermometer suspended from a roof truss reads only 17 °C. The roof and walls of the workshop are made of corrugated iron and are no insulated; when the thermometer is held against the wall, it reads only 5 °C. If the average connective heat transfer coefficient for the suspended thermometer is estimated to be 10 W/(m²·K), what is the true air temperature? (If the thermometer can be modeled as a small gray body in large, nearly black surroundings at 5 °C, the emittance for Pyrex glass is \(\varepsilon = 0.8\), and the Stefan-Boltzmann constant \(\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)\).)

\[T_w = 278 \text{ K}\]

\[T_i = 290 \text{ K}\]

\[T_a\]